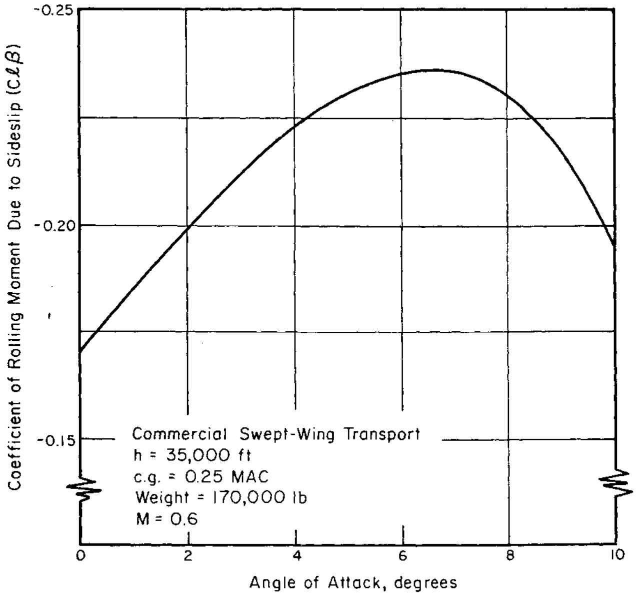

Fig. 6 Typical variation of dihedral effect with angle of attack.

librium sideslip angles required for pronounced coupling are well beyond the normal operating envelope of the class of large swept-wing aircraft which was studied.

Although it has not been established, it is possible that this phenomenon may be of some operational significance for other classes of aircraft, particularly those that may be flown with sustained sideslip, i.e., during a cross-wind landing approach.

References

[1] Etkin, B., *Dynamics of Flight* (John Wiley and Sons, Inc., New York, 1959), Chap. 4, p. 116.

[2] Cole, H. A., Jr., Brown, S. C., and Holleman, E. C., "Experimental and predicted longitudinal and lateral-directional response characteristics of a large flexible 35 degree swept-wing airplane at an altitude of 35,000 feet," NACA Rept. 1330 (1957).

Solution of an Integral Occurring in Propeller Theory

William A. Middleton*

U. S. Naval Ordnance Test Station, Pasadena, Calif.

The integral to be solved arises in the design of propellers and in the consideration of second-order effects in subsonic airfoil theory.[1]

The integral in propeller theory is found in the form

$$M_p = \int_0^\pi \frac{\sin p\,\theta}{\cos\theta - \cos\phi}\,d\theta \qquad (1)$$

where p is an integer and $0 < \phi < \pi$.

Equation (1) satisfies the recursion formula

$$M_p = [(2/p - 1)][1 - \cos(p - 1)\pi] + 2M_{p-1}\cos\phi - M_{p-2} \qquad (2)$$

Received April 27, 1965; revision received July 14, 1965.

* Mathematician, Propulsion Division.

where $p \geq 2$ and

$$M_0 = 0 \qquad M_1 = \ln(1 - \cos\phi) - \ln(1 + \cos\phi) \qquad (3)$$

A closed solution will be derived for an arbitrary integer p. The positive case is considered without loss of generality.

Iteration of the recurrence relation, Eq. (2), using the conditions stated by Eq. (3) gives, for $2 \leq p \leq k$,

$$M_p = M_1 \frac{\sin p\,\phi}{\sin\phi} + 2\sum_{j=2}^{p} \frac{[1 - \cos(j - 1)\pi]}{(j - 1)\sin\phi} \times \sin(p - j + 1)\phi \qquad (4)$$

Consider $p = k + 1$. Substituting Eq. (4) into Eq. (2) yields

$$\left.\begin{aligned} M_{k+1} &= (2/k)(1 - \cos k\pi) + 2M_k\cos\phi - M_{k-1} \\ &= \frac{2}{k}(1 - \cos k\pi) + M_1\frac{\sin(k + 1)\phi}{\sin\phi} + \\ &\quad 2\sum_{j=2}^{k}\frac{[1 - \cos(j - 1]\pi}{(j - 1)\sin\phi}\sin(k - j + 2)\phi \end{aligned}\right\} \qquad (5)$$

where use has been made of

$$\sum_{j=2}^{q}\frac{[1 - \cos(j - 1)\pi]}{(j - 1)\sin\phi}\sin(q - j + 1)\phi = \sum_{j=2}^{q+1}\frac{[1 - \cos(j - 1]\pi}{(j - 1)\sin\phi}\sin(q - j + 1)\phi$$

and

$$\sin(k + 1)\phi - \sin(k - 1)\phi = 2\cos\phi\sin k\phi$$

The quantity q is an arbitrary integer greater than or equal to 2. Now

$$\frac{2}{k}(1 - \cos k\pi) + 2\sum_{j=2}^{k}\frac{[1 - \cos(j - 1)\pi]}{(j - 1)\sin\phi} \times \sin(k - j + 2)\phi = 2\sum_{j=2}^{k+1}\frac{[1 - \cos(j - 1]\pi}{(j - 1)\sin\phi} \times \sin(k - j + 2)\phi \qquad (6)$$

Substituting Eq. (6) into Eq. (5) yields

$$M_{k+1} = M_1\frac{\sin(k + 1)\phi}{\sin\phi} + 2\sum_{j=2}^{k+1}\frac{[1 - \cos(j - 1)\pi]}{(j - 1)\sin\phi} \times \sin(k - j + 2)\phi \qquad (7)$$

Equation (7) is the same form as Eq. (4). Thus, if Eq. (4) is the solution to Eq. (2) for $p = k$, it is also the solution for $p = k + 1$. By the principle of mathematical induction on k, Eq. (4) is the solution to Eq. (2) for all of the integral $p \geq 2$.

Thus it follows that

$$\begin{aligned} M_p &= \int_0^\pi \frac{\sin p\theta}{\cos\theta - \cos\phi}\,d\theta \\ &= M_1\frac{\sin p\phi}{\sin\phi} + 2\sum_{j=2}^{p}\frac{[1 - \cos(j - 1)\pi]}{(j - 1)\sin\phi} \times \sin(p - j + 1)\phi \end{aligned}$$

for integral $p \geq 2$ and

$$M_0 = 0 \qquad M_1 = \ln[(1 - \cos\phi)/(1 + \cos\phi)]$$

Reference

[1] Van Dyke, M. D., "Second-order subsonic airfoil theory including edge effects," NACA Rept. 1274 (1956).